**Proceedings of the**
**Third European Conference on Structural Control, 3ECSC,**
**12-15 July 2004, Vienna University of Technology, Vienna, Austria**

# HEALTH MONITORING OF CABLE SYSTEMS

**Adrián Bekő[1], Jan Holnicki-Szulc[2]**

[1,2] *Institute of Fundamental Technological Research, Polish Academy of Sciences*

The damage detection systems based on an array of piezoelectric transducers sending and receiving strain waves are intensively discussed by researchers recently. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem. The purpose of this paper is to propose an alternative approach to the inverse dynamic analysis problem. Generalizing the so called VDM (Virtual Distortion Method) approach on dynamic problems, a dynamic influence matrix ***D*** concept will be introduced. Pre-computing of the time dependent matrix ***D*** allows decomposition of the dynamic structural response on components caused by external excitation in undamaged structure (the linear part) and on components describing perturbations caused by the internal defects (the non-linear part). In the consequence, analytical formulas for calculation of these perturbations and the corresponding gradients can be derived. Finally, the inverse problem can be solved via a gradient based optimization procedure.

**Keywords:** *damage identification, Virtual Distortion Method, gradient based inverse analysis*

## 1 Introduction

This paper is aimed at addressing a presently much considered problem of damage identification in structural members. A mathematically straightforward formulation for linear cases will be presented for resolving the problem of inverse dynamic analysis minding practical aspects of signal processing.

## 2 Generalization of the Virtual Distortion Method

The Virtual Distortion Method, which belongs to fast reanalysis techniques (cf. [1]) is described in depth in Ref. [2]. The original VDM as applied to structural elements stores the static response of a structure in a so called *influence matrix* which allows fast recomputation of response upon change of geometrical or material characteristics of one or more structural members.

In an arbitrary element we can compute the strain according to

$$\varepsilon_i = \varepsilon_i^{\mathrm{L}} + \varepsilon_i^{\mathrm{R}} = \varepsilon_i^{\mathrm{L}} + \sum_j D_{ij}\hat{\varepsilon}_j\,, \tag{1}$$

where $\varepsilon_i^{\mathrm{L}}$ is the linear component of strain from external loading, $\varepsilon_i^{\mathrm{R}}$ is the residual strain caused by the virtual distortions $\hat{\varepsilon}$ and ***D*** is the influence matrix. The strain influence matrix is a square matrix that collects the strain in element *i* evoked by a unit virtual distortion $\hat{\varepsilon}_j = 1$ on element *j*, where *i,j* = 1… number of elements.

Based on the postulate that an element modeled by a virtual distortion and a modified one have identical general stress and strain fields, we can write the equilibrium of axial forces for a modified bar element and a bar modeled by distortions

$$E_i\hat{A}_i\varepsilon_i = E_iA_i(\varepsilon_i - \hat{\varepsilon}_i)\,, \tag{2}$$

where $A_i$ and $\hat{A}_i$ are the original and modified cross-sectional areas respectively. From Eq. (2) directly follows the definition of the parameter modification vector as

$$\mu_i = \frac{\hat{A}_i}{A_i} = \frac{\varepsilon_i - \hat{\varepsilon}_i}{\varepsilon_i}\,. \tag{3}$$

Next, we generalize these fundaments to the case of dynamical transient analysis (Ref. [4]). In this case we have to introduce the time-dimension to the influence matrix. This means that the virtual distortions will be functions of time and the response of the structure due to these distortions likewise. For our purposes we have to build a three dimensional so called *impulse influence matrix.* An element of this matrix $D_{ij}(t)$ describes the effect on element *i* at time instant *t* of the impulse virtual distortion placed on element *j* in the initial time step. For possible impulse excitation at successive time

[1] Research Scientist, M.Sc., abeko@ippt.gov.pl
[2] Professor, dr hab., holnicki@ippt.gov.pl

instants ($\tau > 0$) matrix $\boldsymbol{D}(t)$ need not be computed, but the following obvious relation may be used

$$\boldsymbol{D}_\tau(t) = \begin{cases} \mathbf{0} & \text{for } t < \tau, \\ \boldsymbol{D}(t-\tau) & \text{for } t \geq \tau. \end{cases} \tag{4}$$

The actual computation of matrix $\boldsymbol{D}(t)$ is conducted by solving the well known equation of motion

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{C}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \mathbf{0}. \tag{5}$$

It is important to notice that from the principle of conservation of momentum

$$\boldsymbol{M}\hat{\boldsymbol{v}}_0 = \hat{\boldsymbol{Q}}\Delta t\,, \tag{6}$$

where $\hat{\boldsymbol{v}}_0$ is the initial velocity vector and $\hat{\boldsymbol{Q}}$ represents the vector of impulse forces, it is advantageous to formulate the effect of time dependent distortions in terms of nonzero initial conditions instead of a time dependent external force. Thus the accompanying initial conditions of Eq. (5) will be

$$\begin{aligned} \boldsymbol{q}(0) &= \mathbf{0}, \\ \dot{\boldsymbol{q}}(0) &= \hat{\boldsymbol{v}}_0\,. \end{aligned} \tag{7}$$

From an initial unit impulse it is easy to simulate any kind of deliberate excitation as a sequence of short impulses as shown in Fig. 1.

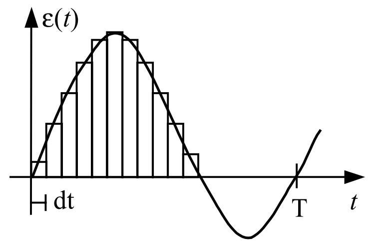


**Figure 1:** Short impulses composing a dynamic excitation

The value of the entire strain for the dynamical case can be computed analogously to Eq. (1) as

$$\begin{aligned} \varepsilon_i(t) &= \varepsilon_i^{\mathrm{L}}(t) + \varepsilon_i^{\mathrm{R}}(t) \\ &= \varepsilon_i^{\mathrm{L}}(t) + \sum_{\tau=0}^{t}\sum_j D_{ij}(t-\tau)\hat{\varepsilon}_j(\tau). \end{aligned} \tag{8}$$

## 3 Modeling Structural Parameter Changes by VDM

As mentioned before virtual distortions may be used to model structural parameters. If we consider all variables time dependent from the definition of the parameter modification vector (Eq. (3)) we obtain the relation for virtual distortion as

$$\hat{\varepsilon}_i(t) = (1-\mu_i)\varepsilon_i(t)\,. \tag{9}$$

Substituting Eq. (8) into Eq. (9) we obtain

$$\begin{aligned} &\sum_{\tau=0}^{t}\sum_j \left[\delta_{t\tau}\delta_{ij} - (1-\mu_i)D_{ij}(t-\tau)\right]\hat{\varepsilon}_j(\tau) \\ &\qquad = (1-\mu_i)\varepsilon_i^{\mathrm{L}}(t), \end{aligned} \tag{10}$$

where $\delta$ denotes the Kronecker delta. Eq. (10) is ineffective for implementation therefore by separating the case of initial time step and transferring the part of summation corresponding to $\varepsilon_i^{\mathrm{R}}$ to the right hand side we derive the following equation for $t = 0$

$$\sum_j \left[\delta_{ij} - (1-\mu_i)D_{ij}(0)\right]\hat{\varepsilon}_j(0) = (1-\mu_i)\varepsilon_i^{\mathrm{L}}(0)\,, \tag{11a}$$

and for $t > 0$

$$\begin{aligned} &\sum_j \left[\delta_{ij} - (1-\mu_i)D_{ij}(0)\right]\hat{\varepsilon}_j(0) \\ &= (1-\mu_i)\left[\varepsilon_i^{\mathrm{L}}(t) + \sum_{\tau=0}^{t-1}\sum_j D_{ij}(t-\tau)\hat{\varepsilon}_j(\tau)\right]. \end{aligned} \tag{11b}$$

The presented Eq. (11) is a system of linear equations with virtual distortions $\hat{\boldsymbol{\varepsilon}}$ as unknowns. Solving the set of Eqs. (11) the virtual distortions for each time step will be known, which allows us the computation of the complete strain vector $\boldsymbol{\varepsilon}(t)$ based upon the parameter modification vector $\boldsymbol{\mu}$.

## 4 Sensitivity Analysis of a Structure

Let us assume that the analyzed problem is an optimization task with a given objective function in the form

$$f_{obj} = \sum_{t\in T} F_t(\boldsymbol{\mu}) \quad \text{where} \quad F_t(\boldsymbol{\varepsilon}(t,\boldsymbol{\mu})). \tag{12}$$

In this case it is possible to derive the analytical formula for the gradients of the objective function of the form

$$\frac{\partial f_{obj}(\boldsymbol{\mu})}{\partial \mu_s} = \sum_\alpha\sum_{t\in T}\frac{\partial F_t(\boldsymbol{\varepsilon})}{\partial \varepsilon_\alpha}\frac{\partial \varepsilon_\alpha(t,\boldsymbol{\mu})}{\partial \mu_s}. \tag{13}$$

If we consider our structural parameter modeling based on VDM as given in Eq. (11), differentiating this equation with respect to $\boldsymbol{\mu}$ will give us the unknown distortion gradients in terms of the following equation

$$\begin{aligned} &\sum_j \left[\delta_{ij} - (1-\mu_i)D_{ij}(0)\right]\frac{\partial \hat{\varepsilon}_j(t,\boldsymbol{\mu})}{\partial \mu_s} = -\delta_{is}\varepsilon_i(t,\boldsymbol{\mu}) \\ &\qquad + (1-\mu_i)\sum_{\tau=0}^{t-1}\sum_j D_{ij}(t-\tau)\frac{\partial \hat{\varepsilon}_j(\tau,\boldsymbol{\mu})}{\partial \mu_s}, \end{aligned} \tag{14}$$

which again represents a set of linear equations.

## 5 Damage Identification

Our goal is to detect possible defect locations and their intensities within a cable structure. The methodology is based upon the assumption of the existence of a net of piezo-transducers mounted to the structure. One of them will act as a generator of elastic strain waves in the structure, while another one (or more) will act as a sensor(s) collecting the transmitted signal. The identification relies on the phenomenon that the propagated signal is changed due to potential damages within the structure. For this case our virtual distortions $\hat{\varepsilon}$ become strains externally induced by the piezo-electric transducers.

Now we formulate the inverse dynamic problem where the damage sizes and locations are represented by vector $\mu$ which in the present context we shall call *defect vector*. Presuming that we have structural response data from measurement, we can formulate the objective function as

$$f_{ident} = \sum_{\alpha}\sum_{t}\left[\varepsilon_{\alpha}(t,\mu) - \varepsilon_{\alpha}^{\mathrm{M}}\right]^2, \qquad (15)$$

where $\varepsilon_{\alpha}^{\mathrm{M}}$ are the strain responses at locations $\alpha$ to the known excitation generated in a specified location, and $\varepsilon_{\alpha}(t,\mu)$ are the strains based on numerical modeling (FEM+VDM). The most probable defect identification leads to the minimization problem: **min** $f_{ident}$, with $\mu_i$ as design variables. To achieve this, a gradient based approach can be applied. Realizing that the gradients of $\varepsilon_{\alpha}(t,\mu)$ are only dependent on their residual part (their linear part remaining constant during the optimization), we can obtain the analytical gradients of the objective function according to Eq. (13) in the form

$$\frac{\partial f_{ident}}{\partial \mu_s} = 2\sum_{\alpha}\sum_{t}\left[\varepsilon_{\alpha}(t,\mu) - \varepsilon_{\alpha}^{\mathrm{M}}\right]^2 \frac{\partial \hat{\varepsilon}_j(t,\mu)}{\partial \mu_s}, \qquad (16)$$

where the gradients of distortions $\partial\hat{\varepsilon}_j/\partial\mu_s$ may be computed by solving the set of linear Eqs. (14).

## 6 Numerical Example and Remarks

In the following example we present the functionality of the proposed method. The cable structure under consideration has the geometrical and physical properties given in Fig. 2 and Table 1 respectively. The length of the considered cable was 1.5m with a diameter of 0.5cm. At this stage the cable was modeled as a two dimensional slender beam structure with all DOFs constrained at its upper end. The structure was discretized into 30 finite elements of equal length of 5cm. In this phase the winding of an actual steel cable was neglected and its material characteristics were assumed as those of standard steel. The simulated piezo-actuator was situated on element 4 (symbol **A** in Fig. 2) and the analogous sensor was located on element 24 (symbol **S**). A possible damage area of 8 elements was assumed between the actuator and sensor, which included elements 5 to 12.

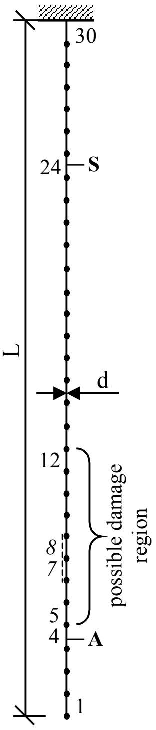


**Table 1:** Geometrical and material characteristics

| | |
|---|---|
| *geometry* | |
| length | L=1.5 m |
| diameter | d=0.5 cm |
| *material* | |
| Young's mod. | E=210 MPa |
| Poisson's ratio | μ=0.3 |
| density | ρ=7800 kg.m$^{-3}$ |
| *additional* | |
| actuator position | **A** – element 4 |
| sensor position | **S** – element 24 |

**Table 2:** Position and size of numerically provoked damage

| | |
|---|---|
| element 7 | 40% |
| element 8 | 30% |

**Figure 2:** Geometry of cable structure

The structure was excited by a sinusoidal load of frequency $f$=610Hz which corresponds to period of loading $T$=1.64 milliseconds. The complete time of the transient analysis was assumed 4$T$.

For the verification purpose of the damage identification a dummy structure with numerically simulated damages was assumed to imitate a possibly measured experimental response. In the dummy structure the damage was modeled in terms of stiffness reduction according to Table 2. Elements 7 and 8 were weakened by 40 and 30% respectively.

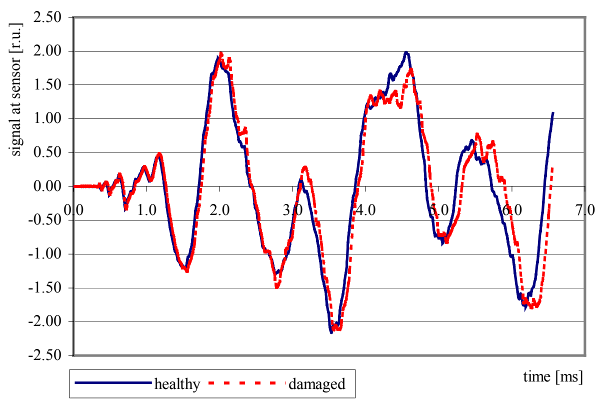


**Figure 3:** Comparison of signals (at sensor) of healthy and damaged structure

In Fig. 3 the responses of a healthy and damaged structure are displayed in terms of strain measure as picked up by the sensor. The clear difference of signals permits us to expect a feasible solution of the identification process.

During the identification process our defect vector $\mu_i$, which represents the design variables must obviously satisfy the constrains: $0 \le \mu_i \le 1$. For resolution of the constrained optimization task a modification of the *gradient projection method* based on Ref. [3] was implemented. The identification procedure is started with all values of the defect vector $\mu_i$ = 1, which corresponds to no stiffness loss in the elements. The course of the optimization can be seen in Figs. 4 and 5. The values of the objective function $f_{ident}$ in successive iteration steps are pictured in Fig. 4, which shows a successful convergence.

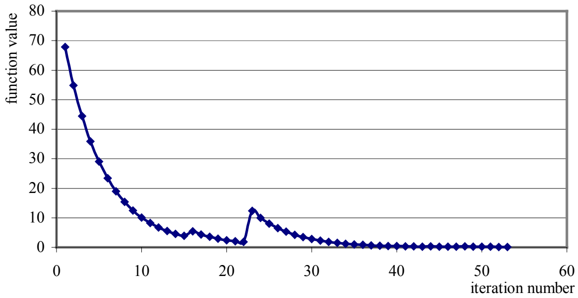


**Figure 4:** Evolution of objective function ($f_{ident}$) values in successive iterations

The progression of the design variable values is illustrated in Fig. 5. The values of $\mu_i$ are represented in terms of remaining stiffness.

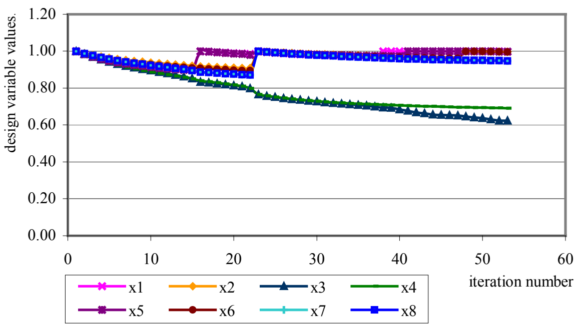


**Figure 5:** Evolution of design variable values in successive iterations

Table 3 sums up the results of the optimization. It can be seen that the damage in elements 7 and 8 was identified successfully. The relative error presents 3.7% in case of element 7 and 1.2% for element 8. Disregarding numerical inaccuracy the remaining elements may be considered undamaged.

**Table 3:** Resulting stiffness of elements 5-12

| $nr_e$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|
| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mu_i$ [%] | 99.8 | 99.7 | 62.3 | 69.2 | 99.8 | 99.6 | 94.8 | 99.3 |

However, we have to note that the algorithm resulted in a 5.2% stiffness reduction in element 11 which is false damage. It must be said that for a different structure, actuator and sensor locations or potential damage region higher number or more significant false damage locations may be encountered. This "side effect" of the technique must be treated in further research. Multi-sensor formulation of the problem may be helpful in this case.

## 7 Conclusion

In the presented paper a methodology for structural damage identification is described. A concurrent approach to the widely used algorithms based on soft computing methods is introduced, which is based on analytical computation of the objective function gradients. Thus the optimization may be solved by a classical gradient based method. The numerical example shows that the algorithm is capable of recognizing structural damages resulting in relatively excessive loss of stiffness. More subtle damages may be identified by the method, however further research and sophistication of the methodology is required, along with lowering the computational expense of the algorithm. Additionally, an experimental verification of the method is planned.


## Acknowledgements

This project was carried out within the *Research Training Network* - 6th Framework Programme (HPRN-CT-2002-00284), which is thankfully acknowledged for its financial support. Furthermore, we acknowledge the research program 1/9360/02 granted by the Scientific Grant Agency of the Slovak Ministry of Education.

The presented paper is the final degree work done during the course of Smart Technology Expert School, operating in the Institute of Fundamental Technological Research, Warsaw, Poland.


## References


[1] Akgün M. A., Garcelon J. H., Haftka R. T., Fast Exact Linear and Non-linear Structural Reanalysis and the Sherman-Morrison-Woodbury Formulas, *International Journal for Numerical Methods in Engineering*, Vol. 50, pp. 1587-1606, 2001.

[2] Holnicki-Szulc J., Gierliński J. T., *Structural Analysis, Design and Control by the Virtual Distortion Method*, J. Wiley & Sons, Chichester, U.K., 1995.

[3] Haftka R. T., Gürdal Z., *Elements of Structural Optimization*, Kluwer Academic Publishers, Dortrecht, Boston and London, 1992.

[4] Zieliński T. G., Virtual Distortions Applied to Structural Modelling and Sensitivity Analysis, *Proceedings SMART'03 AMAS Workshop on "Smart Materials and Structures", Jadwisin, September 2-5, 2003*, Warsaw, 2004 (CD edition).